Examples and exercises For River engineering

Chapter One

1. Development of the boundary layer flow.

Given flow velocity $U = 1m/s$ and water depth $h = 10m$

Wanted 1) x value where the boundary layer flow starts to fulfil the whole depth

2) type of the boundary layer flow

Solution Based on the expression for turbulent boundary layer flow

$$x\big|_{\delta=h} = \left(\frac{(U/\nu)^{0.25}}{0.4^{0.8}}\ h^{1.25}\right) = \left(\frac{(1/10^{-6})^{0.25}}{0.4^{0.8}}\ 10^{1.25}\right) = 1171\ m$$

$$Re_x = \frac{U\ x}{\nu} = \frac{1\ \times\ 1171}{10^{-6}} = 1.171\ \times\ 10^9 > 5 \times 10^5 \quad \text{turbulent}$$

Comment The example demonstrates that the flow in open channels is always a turbulent boundary layer flow.

2. Chezy`s coefficient and bottom shear stress

Given A project is to be located at the water depth h=5 m in Kattegat strait. The measured tidal current velocity is U=1 m/s (Havnecon a/s). The sediment size is d_{90}=0.15 mm (Danish Geotechnic Institute). It is estimated that the height of sand ripples is app. 10 cm.

Wanted 1) Bottom shear stress τ_b, when there are sand ripples with height of app. 10 cm on the bed

2) Bottom shear stress τ_b, if the flow is hydraulically smooth.

Solution 1) When there are sand ripples on the bed

Bed roughness $k_s \approx 0.75 \times (\text{height of sand ripple}) = 0.075\ m$

Chézy coefficient $C = 18\ \log\left(\frac{12\ h}{k_s}\right) = 18\ \log\left(\frac{12\times 5}{0.075}\right) = 52.3\ \sqrt{m}/s$

Friction velocity $u_* = \frac{U}{C}\ \sqrt{g} = \frac{1}{52.3}\ \sqrt{9.8} = 0.06\ m/s$

Bed shear stress $\tau_b = \rho\ u_*^2 = 1000 \times 0.06^2 = 3.6\ N/m^2$

The flow is hydraulically rough as

$$\frac{u_*\ k_s}{\nu} = \frac{0.06 \times 0.075}{10^{-6}} = 4500 > 70$$

3. Consider the velocity profile of a river shown in the table at a discharge $Q = 24$ m3/s, a width $W = 52$ m, and depth 1.5 m. The friction slope is about 10 cm/km and the bed material is very coarse gravel. Assume steady uniform turbulent flow, kinematic viscosity $\nu = 10$-6 m2/s, water density $\rho = 1000$ kg/m3, and $\kappa = 0.4$. From measured velocity profile estimate the following parameters:
(a) shear velocity; (b) bed shear stress (c) elevation of zero velocity (e) Chezy's coefficient

given
$Q = 24\ m^3/s$
W= 52m s= 10cm/km = 10^{-4}
h= 1.5m $\nu = 10^{-6}\ m^2/s$
K=0.4 $\rho = 1000\ kg/m^3$

a. shear velocity,

$$u_* = \sqrt{g\ R\ S}$$

$U_* = (9.81*1.5*10^{-4})^{0.5}$

= 0.038m/s

b. bed shear stress,

$$\tau_b = \rho\ g\ R\ S$$

$\tau_b = 1000*9.81*1.5*10^{-4}$

$= 1.47 N/m^2$

c. elevation of zero velocity, Z_0

$$u = \frac{u_*}{k}(\ln z - \ln z_0)$$

$$lnZo = lnZ - \frac{U(Z)*K}{U*} = ln1.5 - \frac{0.45*0.4}{0.038} = -4.33$$

Zo=0.013m

d. viscous sublayer thickness

$$\delta = 11.6\frac{\upsilon}{u_*}$$

$$= \frac{11.6 * v}{U *} = \frac{11.6 * 10 - 6}{0.038} = 3.05 * 10 - 4$$

e. Chezy's coefficient

$\overline{U} = C\sqrt{RI}$, R = A/P = $\frac{52*1.5}{2*1.5+52} = 1.42m$

$\overline{U} = \frac{Q}{A} = \frac{24}{52*1.5} = \frac{0.31m}{s}$

$$C = \frac{U}{\sqrt{Rs}} = \frac{0.31}{\sqrt{1.42 * 10 - 4}} = 26.03$$

Chapter Two

4. What is the sediment size corresponding to beginning of motion when the shear velocity $u_* = 0.1$ m/s?

Solution The shear stress corresponding to $u_* = 0.1$ m/s is

$$\tau = \rho u_*^2 = (1000)\,(0.1)^2 = 10 \text{ N/m}^2$$

$$\frac{\tau_c}{(\rho_s - \rho) g d_s} = 0.047$$

then we get

$$d_s = 1.31 \text{ cm}$$

5. Given the stream slope $S_0 = 10^{-3}$, at what flow depth would coarse gravel enter motion?

Solution For coarse gravel, we have $d_s = 16 - 32$ mm. Now

$$\frac{\tau_c}{(\rho_s - \rho) g d_s} = \frac{\rho g h S_0}{(\rho_s - \rho) g d_s} = \frac{h S_0}{(G-1)\, d_s} = 0.047$$

then

$$h = \frac{0.047\,(G-1)\,d_s}{S_0} = \frac{0.047\,(1.65)\,(16 \times 10^{-3})}{10^{-3}} = 1.24 \text{ m}$$

6. PROPOSED DAM AND RESERVOIR CONSTRUCTION PLAN

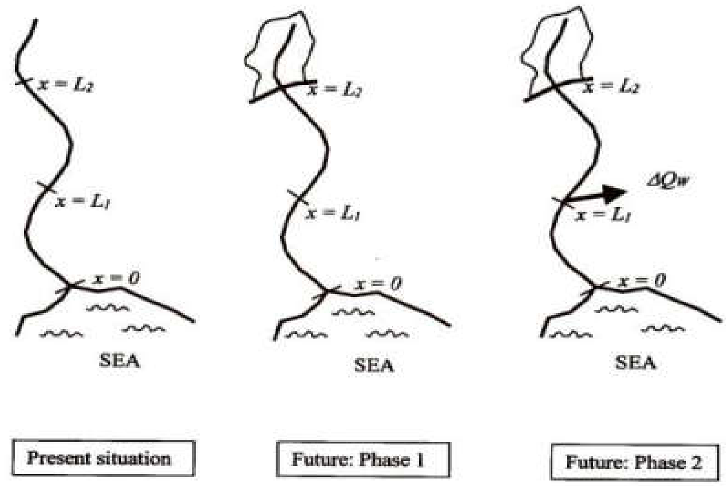

Figure 1: Dam and Reservoir Construction Plan on a River

NUMERICAL DATA PROVIDED:

Distance from x = 0, computed along the river:

L1 = 54km, L2 = 81km; River width: B = 800m

Longitudinal bed slope: $i = 1*10^{-4}$

Average water depth at the river mouth; present situation:

Representative water depth, h_m=8.75m

Discharges:

Present situation (10 months per year); $Q_{WL}=3600m^3/s$

Present situation (2 months per year); $Q_{WH}=12000m^3/s$

Future phase 1& Phase 2: (constant discharge) $Q_{WM}=5000m^3/s$

Future only phase 2: (Extracted discharge); $\Delta Q_W=500m^3/s$

Hydraulic roughness: $C=50m^{1/2}/s$ (Constant)

Sediment:

Uniform sand with: D=0.5mm; Fall velocity (ω_s)= 0.06m/s

Sediment density: $\rho_s=2650kg/m^3$

For the above given data select appropriate sediment transport formula and determine the total anuual (per year) sand treansport of the river (assume normal flow for the two discharges).

Solution

1.1. Sediment Transport

Checking the validity of the Engelund and Hansen or Meyer- Peter and Muller formula for the given situation;

Engelund and Hansen

- $\frac{\omega_s}{u_*} < 1$
- $0.19 < D_{50} < 0.93\text{mm}$
- $0.07 < \theta < 6$

Meyer-Peter and Muller formula:

- $\frac{\omega_s}{u_*} > 1$
- $D_m > 0.4\text{mm}$
- $\mu\theta < 0.2$

Engelund and Hansen formula:

$$q_{st} = m(u - u_c)^b = \left\langle \frac{1}{(12 or 20)C^3\Delta^2 D_{50}\sqrt{g}} \right\rangle u^5 \Rightarrow m = \frac{1}{(12 or 20)C^3\Delta^2 D_{50}\sqrt{g}} \& b = 5$$

Meyer-Peter and Muller formula:

$$q_{sb} = (8 or 13.3)\sqrt{\Delta g D_m{}^3}\sqrt{(\mu\theta - 0.047)^3} \Rightarrow b = \frac{3\mu\theta}{\mu\theta - 0.047}$$ Here m is implicitly expressed.

Where; C=Overall Chezy coefficient

$$\Delta = \frac{\rho s - \rho w}{\rho w}; \quad \mu = \sqrt{(C / C90)^3}$$

$$\theta = \frac{u^2}{C^2 \Delta D_m}; \; C_{90} = 18\log_{10}\left(\frac{12h}{D_{90}}\right)$$

Where Δ relative density of sediment

$$\Delta = \frac{(\rho_s - \rho_W)}{\rho_W}$$

Hydraulic parameters are computed as follows:

$$\text{Normal depth } (h_n) = \left(\frac{Q^2}{B^2C^2 i}\right)^{\frac{1}{3}}, \text{Critical depth } (h_c) = \left(\frac{q^2}{g}\right)^{\frac{1}{3}},$$

$$\text{Specific discharge } (q) = \frac{Q}{B}, \text{ Flow velocity } (u) = C\sqrt{Ri} = C\sqrt{hi}, \text{ for } (B >> h).$$

$$\text{shear velocity } (u_*) = \sqrt{\frac{\tau}{\rho}} = \frac{u\sqrt{g}}{C} = \sqrt{ghi}, \text{ Shield's parameter } (\theta) = \frac{u^2}{C^2\Delta D}$$

$$\text{Froude number } (F) = \frac{u}{(gh)^{\frac{1}{2}}}, \ (C_{90}) = 18\log_{10}\left(\frac{12h}{D_{90}}\right)$$

$$\text{Bed form factor } (\mu) = \left(\frac{C}{C_{90}}\right)^{\frac{3}{2}}$$

The results for each parameter are tabulated below after manipulated in excel:

Table 1: Hydraulic parameters of each flow condition

Discharge (m^3/s)		Months	q_w (m^2/s)	h_n (m)	h_c (m)	u(m/s)	Fr
Q_{WL}	3600	10	4.5	4.33	1.27	1.04	0.16
Q_{WH}	12000	2	15	9.65	2.84	1.55	0.16
Q_{WM}	5000	12	6.25	5.39	1.59	1.16	0.16
Q_{WM} - ΔQ_W	4500	12	5.625	5.02	1.48	1.12	0.16
$\Delta Q_W = 500$							

Parameters related to sediment transport:

Discharge (m^3/s)		D_m	u_* (m/s)	C_{90}	ω_s/u_*	μ	θ	$\mu\theta$	**E&H**	**M, P & M**
Q_{WL}	3600	***0.5***	0.065	90.295	***0.921***	0.412	***0.5245***	***0.216***	✓	X
Q_{WH}	12000	***0.5***	0.097	96.569	***0.617***	0.373	***1.1703***	***0.436***	✓	X
Q_{WM}	5000	***0.5***	0.073	92.007	***0.825***	0.401	***0.6529***	***0.262***	✓	X
Q_{WM} - ΔQ_W	4500	***0.5***	0.070	91.458	***0.855***	0.404	***0.6086***	***0.246***	✓	X

The four highlighted columns in table 2 are checked for the validity of Engelund and Hansen, and Meyer-Peter & Muller formulas based on their respective criteria.

Checking the validity of Engelund & Hansen formula:

- ✓ Fall velocity: $\frac{\omega_s}{u_*} < 1$; ***satisfied for every discharge***
- ✓ Grain size: 0.19< 0.5 <0.93mm; ***satisfied***
- ✓ Shields parameter: $0.07 < \theta < 6$; ***satisfied for every discharge***

Checking the validity of Meyer-Peter & Muller formula;

- ✓ Fall velocity: $\frac{\omega_s}{u_*} > 1$; ***Never satisfied in any discharge condition***
- ✓ Grain size: 0.5 > 0.4mm; ***Satisfied***
- ✓ Shields parameter: $\mu\theta < 0.2$; ***Never satisfied***

As shown in Table. 2, only Engelund and Hanson method is valid for each discharges satisfying the ***three criteria at a time***. Therefore Engelund and Hansen formula is used to compute the annual sediment transport capacity of the river.

Annual Sediment Transport;

$$q_{st} = m(u - u_c)^b = \left\langle \frac{1}{(12 or 20)C^3\Delta^2 D_{50}\sqrt{g}} \right\rangle u^5 \Rightarrow m = \frac{1}{(12 or 20)C^3\Delta^2 D_{50}\sqrt{g}} \;\&\; b = 5$$

$$q_{st} = \left\langle \frac{1}{(12)C^3\Delta^2 D_{50}\sqrt{g}} \right\rangle u^5 \text{ ….. [Including pores]}$$

- ✓ **For the 10 months flow of 3600m^3/s, and velocity u = 1.04m/s**

$$q_{st\text{-}10months} = \left\langle \frac{1}{(12)*50^3*\left(\frac{2650-1000}{1000}\right)^2*0.0005\sqrt{9.81}} \right\rangle 1.04^5 = \underline{\mathbf{1.9024*10^{-4} m^3/s/m}}$$

- ✓ **For the 2 months flow of 12000m^3/s, and velocity u = 1.55m/s**

$$q_{st\text{-}2months} = \left\langle \frac{1}{(12)*50^3*\left(\frac{2650-1000}{1000}\right)^2*0.0005\sqrt{9.81}} \right\rangle 1.55^5 = \underline{\mathbf{1.354*10^{-3} m^3/s/m}}$$

Yearly;

$$q_{st} = q_{st\text{-}10months} * (10/12) * 365 * 24 * 60 * 60 + q_{st\text{-}2months} * (2/12) * 365 * 24 * 60 * 60$$
$$q_{st} = 1.9024 * 10^{-4} * (10/12) * 365 * 24 * 60 * 60 + 1.354 * 10^{-3} * (2/12) * 365 * 24 * 60 * 60$$

$$q_{st} = 4,999.521 + 7118.612 = \underline{\underline{12,118.133 m^3 / yr / m}}$$

$$Q_{st} = q_{st} * B = 12118.133 * 800 = \underline{\underline{9,694,506.221 m^3 / yr}}$$

If pores are not considered the E&H formula would have 20 in the denominator instead of 12, therefore please calculate the example using the criteria without pores.

7. 1. A sediment transporting river with an approximately rectangular cross section has a mean flow of ***125 m^3/s***. The average width and depth of the river is ***20 m*** and ***5 m***, respectively. Bed material median grain size is ***2 mm***, ***d_{90} = 8 mm***, and slope ***I = 0.1m/km***. The water temperature is ***20^oC***. Other data are: ***ρ_s = 2650 kg/m^3***, ***ρ = 10^3 kg/m^3***, ***ν = 10^{-6} m^2/s***. Use $\omega_s = 1.48$.

Calculate the bed load sediment transport along the bottom.

Specific discharge $(q) = \frac{Q}{B}$, Flow velocity $(u) = C\sqrt{Ri} = C\sqrt{hi}$, for $(B >> h)$.

shear velocity $(u_*) = \sqrt{\frac{\tau}{\rho}} = \frac{u\sqrt{g}}{C} = \sqrt{ghi}$, Shield'sparameter $(\theta) = \frac{u^2}{C^2 \Delta D}$

Froude number $(F) = \frac{u}{(gh)^{\frac{1}{2}}}$, $(C_{90}) = 18\log_{10}\left(\frac{12h}{D_{90}}\right)$

Bed form factor $(\mu) = \left(\frac{C}{C_{90}}\right)^{\frac{3}{2}}$

<u>Engelund and Hansen</u>

- $\frac{\omega_s}{u_*} < 1$
- $0.19 < D_{50} < 0.93mm$

<u>Meyer-Peter and Muller formula:</u>

- $\frac{\omega_s}{u_*} > 1$
- $D_m > 0.4mm$

- $0.07 < \theta < 6$
- $\mu\theta < 0.2$

$$q_{sb} = (8 or 13.3)\sqrt{\Delta g D_m{}^3}\sqrt{(\mu\theta - 0.047)^3} \Rightarrow b = \frac{3\mu\theta}{\mu\theta - 0.047}$$

Where; C=Overall Chezy coefficient

$$\Delta = \frac{\rho s - \rho w}{\rho w};\ \mu = \sqrt{C / C90)^3}$$

$$\theta = \frac{u^2}{C^2 \Delta D_m};\ C_{90} = 18\log_{10}\left(\frac{12h}{D_{90}}\right)$$

The given river has a rectangular cross section

$A = h*w = 5*20 = 100m^2$

$Q = A\bar{u}$, $\bar{u} = Q/A$ where $\bar{u}$= depth average flow velocity

$\bar{U} = 125/(5*20) = 1.25 m/s.$

$R=A/P = 100m^2/(20+5+5) = 3.33$

$$\text{shear velocity } (u_*) = \sqrt{\frac{\tau}{\rho}} = \frac{u\sqrt{g}}{C} = \sqrt{ghi} = \sqrt{9.81 * 5 * 0.0001} = 0.07 m/s$$

$C=\bar{u}/(RI)^{0.5} = 1.25/(3.33*10^{-4})^{0.5} = 68.49 m^{1/2}/s$

$$C_{90} = 18\log_{10}\left(\frac{12h}{D_{90}}\right)$$

$C_{90} = 18\log(12h/d_{90}) = 18\log(12*3.33/(8*10^{-3})) = 66.57 m^{1/2}/s$

$$\text{Bed form factor } (\mu) = \left(\frac{C}{C_{90}}\right)^{\frac{3}{2}}$$

$\mu = (C/C_{90})^{1.5} = (68.49/66.57)^{1.5} = 1.04$

$\frac{\omega_s}{u_*} = 1.48/0.07 = 21.14$

$$\text{Shield'sparameter } (\theta) = \frac{u^2}{C^2 \Delta D}$$

Where relative density of sediment

$$\Delta = \frac{(\rho_s - \rho_W)}{\rho_W}$$

= (2650-1000)/1000 =1.65

$$\Theta = \frac{(1.25\text{m/s})^2}{68.49^2 * 1.65 * 0.002} = 0.1$$

$\mu\Theta$ = 1.04 *0.1= 0.105

Discharge (m^3/s)		D_m	u* (m/s)	C_{90}	ω_s/u_*	μ	θ	μθ	E& H	M, P & M
Q_{WL}	3600	***2mm***	0.07	66.57	21.14	1.04	***0.1***	0.105	x	OK!

<u>Meyer-Peter and Muller formula validity check</u>

$\frac{\omega_s}{u_*} = 21.14 > 1$,Satisfied!

$D_m = 2\text{mm} > 0.4mm$,Satisfied!

$\mu\theta = 0.105 < 0.2$,Satisfied!

$$q_{sb} = (8 or 13.3)\sqrt{\Delta g D_m{}^3}\sqrt{(\mu\theta - 0.047)^3} \Rightarrow b = \frac{3\mu\theta}{\mu\theta - 0.047}$$

Without pores;

$$q_{sb} = 8\sqrt{1.65 * 9.81 * (0.002m)^3}\sqrt{(0.105 - 0.047)^3}$$

$$q_{sb} = 8 * 3.6 * 10^{-4} * 0.01397$$

$$q_{sb} = 4.023 * 10^{-5}\ \text{m}^2/\text{s}/\text{m}$$

$$q_{sb} = 4.023 * \frac{10^{-5}\text{m}^2}{\text{s}} * 1 * 365 * 24 * 60 * 60 = 1268.7\ \text{m}^3/\text{year}/\text{m}$$

$$Q_{sb} = B * q_{sb} = 20m * 1268.7\ \text{m}^3/\text{year}/\text{m}$$

$$Q_{sb} = 25373.87\ \text{m}^3/\text{year}$$

Including pores;

$$q_{sb} = 13.3\sqrt{1.65 * 9.81 * (0.002m)^3}\sqrt{(0.105 - 0.047)^3}$$

$$q_{sb} = 13.3 * 3.6 * 10^{-4} * 0.01397$$

$$q_{sb} = 6.688 * 10^{-5}\ \text{m}^3/\text{s}/\text{m}$$

Chapter five

8. **Example 8.1.** *The following hydraulic data pertains to a bridge site of a river.*

Maximum discharge = 6,000 cumecs.

Highest flood level = 104.00 m.

River bed level = 100.00 m.

Average diameter of river bed material = 0.10 mm.

Design and sketch Bell's Bunds including the launching apron to train the river. Assume plentiful availability of boulders near the site.

Solution. The waterway between two guide bunds, planned as per Fig. 8.13 may be evaluated as follows :

The Lacey's regime waterway = clear waterway

$$P = 4.75\sqrt{Q} = 4.75\sqrt{6000}\text{ m} = 368\text{ m}.$$

Allowing 20% extra for piers, etc., the net spacing between the two guide bunds at the bridge site = $1.2 \times 368 = 440$ m

Hence $L = 440$ m.

∴ The length of the guide bund upstream of bridge (From Fig. 8.16)

$$= 1.25 \times 440 = 550\text{ m}.$$

The length of the guide bund downstream of bridge

$$= 0.25\,L = 0.25 \times 440 = 110\text{ m}.$$

The radius of the curved head (upstream portion) may be kept

$$= 0.45\,L = 0.45 \times 440 = \text{say } 194\text{ m}.$$

The u/s end of guide bund may therefore, be curved by 130° (between 120 to 145°) with a radius of 194 m.

The d/s end of the guide bund may, therefore, be curved in such a way as to make an angle of 60°, as shown in Fig. 8.22 (*a*).

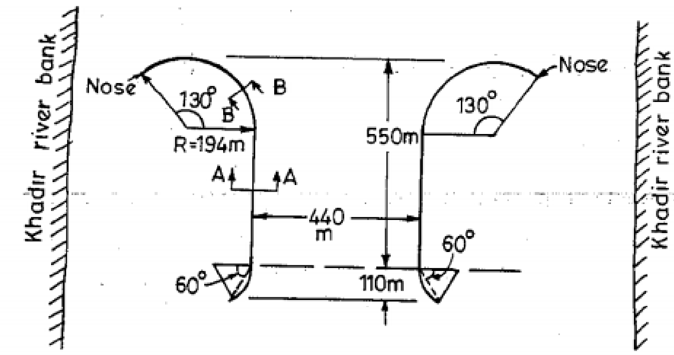

Fig. 8.22 (*a*) Plan of guide bunds for Example 8.1.

Cross-sections of the guide bunds

The given HFL at bridge site = 104.00 m.

Assuming a free-board of 1.5 m, and nil value of afflux, and ignoring velocity head*, we have the

Top level of guide bund = 104.00 + 1.5 = 105.5.

To be more safe and making an allowance for future settlement etc. ; let us adopt the top level of bund as 106.00 m.

Now, height of the bund above river bed level

= 106.00 − 100.00 = 6.00 m.

Assuming the top width of the bund as 5 m, and side slopes as 2 : 1, we have the required sections of the bund as shown in Figs. 8.22 (*b* & *c*).

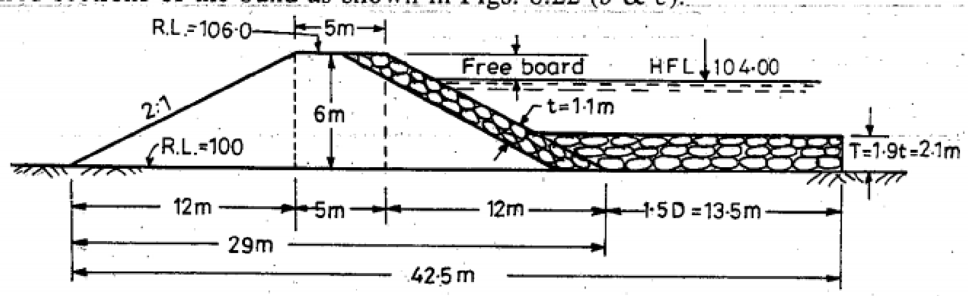

Fig. 8.22 (*b*) Cross-section (*AA*) of the straight reach of Bell's Bund.

The stone pitching and an apron must be provided on the slope on water side for the entire length of the bund ; however, the rear side will also be pitched for curved portions of the bund.

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*Vel. head $= \dfrac{V^2}{2g}$

where , $V = \dfrac{6000}{PR} = \dfrac{6000}{368 \times 10.36} = 1.57$ m/.sec.

(*R* is calculated afterwards)

∴ Vel. head $= \dfrac{(1.57)^2}{2 \times 9.81} = 0.13$ m.

Design of stone pitching and apron

The thickness of stone pitching on the side (t) is given by Eq. (8.4). as :

$$t = 0.06\,Q^{1/2} = 0.06 \times (6000)^{1/2}$$
$$= 0.06 \times 18.2 = 1.09 \text{ m} \text{ ; say } 1.1 \text{ m}$$

The thickness of apron is given by eqn. (8.6)

$$= 1.9\,t = 1.9 \times 1.1 = 2.09 \text{ m} \text{ ; } \mathbf{say\ 2.1\ m.}$$

The length of the apron is given by 1.5 D, where D for *the straight reaches* of the guide bund is given as :

$$D = 1.25\,R - y$$

$$\text{where } R = 0.47\left(\frac{Q}{f}\right)^{1/3} \qquad ...(8.6)$$

where f is the Lacey/s silt factor given by Eqn. (4.24) as, $f = 1.76\sqrt{d_{mm}}$

where, d_{mm} is the av. particle dia in mm = 0.1 mm (given)

$$f = 1.76\sqrt{0.10} = 1.76 \times 0.322 = \mathbf{0.556}.$$

$$\therefore\ R = 0.47\left(\frac{6000}{0.556}\right)^{1/3} = 0.47\,(10780)^{1/3}$$
$$= 0.47 \times 22 = \mathbf{10.36\ m.}$$

y = Depth of water above river bed level

$$= 104.0 - 100.00 = \mathbf{4.00\ m.}$$

$$\therefore \quad D = 1.25 \times 10.36 - 4.00 = 13 - 4.0 = \mathbf{9.0\ m.}$$

Length of apron = $1.5\,D = 1.5 \times 9.0 = 13.5$ m.

For the curvilinear transition portions of the guide bund, however, D is given by

$$D = 1.5\,R - y = 1.5 \times 10.36 - 4.00 = 15.54 - 4.00 = 11.54\text{; say } 12 \text{ m.}$$

Hence, the length of apron in the curved portions will be = 1.5×12 m = 18 m. Pitching and apron in these curved portions shall be provided on both sides, as shown in Fig. 8.22 (c).

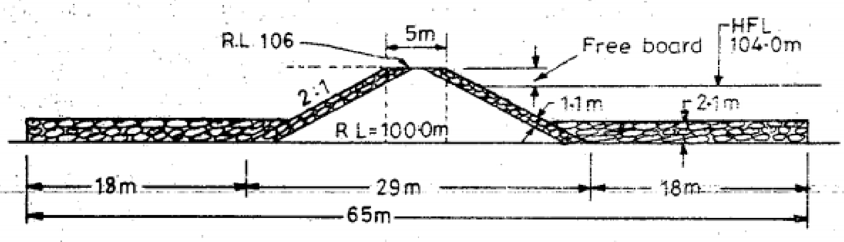

Fig. 8.22 (c) Cross-section (BB) of the curved portions of the Bell's bund.

The length of apron at noses should be increased to 1.5 D (where $D = 2.24 \times 10.36 - 4.00 = 19.36$) *i.e.* say 30 m.